Postdoctoral fellowship 2023

Study of the existence and uniqueness of solutions of the non-incremental symplectic Brezis-Ekeland-Nayroles variational principle with bipotential

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Type of funding:

The postdoctoral fellowship is financed by the French Research National Agency (ANR) in the frame of the research project BigBen. It starts as soon as possible for a 12 months duration.

Laboratory:

The postdoctoral fellowship runs amongst the research team COREFoU (Comportement et mécanismes d'endommagement et de fatigue) of the Laboratoire de Mécanique, Multiphysique, Multiéchelle (LamCube, UMR CNRS 9013)

Context:

Our aim is to develop innovative non-incremental variational methods to address from the mathematical and numerical viewpoints challenges today in structural mechanics in the domain of the couplings and interactions between phenomena such as the dynamics, the plasticity, the frictional contact, the fracture mechanics and the damage.

The non associative laws cannot be represented by a convex potential. In contrast, we showed that they can be modeled thanks to a bipotential, a function of 2 dual variables, biconvex. The bipotential approach leads to an extensive generalization of the calculus of variation. The Brezis-Ekeland-Nayroles principle is a non-incremental space-time variational principle. In [Buliga 2016], we extended it to the dynamics by introducing the concept of symplectic subdifferential and the SBEN principle of minimum.

Subject:

The bipotentials are theoretical tools to modelize the non-associated constitutive laws and to study the existence ans uniqueness of solutions of variational problems [Laborde 2008, Matei 2011, Matei 2013]. In [Buliga 2012], we proposed a rigorous variational incremental formulation of the bipotential for elastoplastic structures problems and we proved that if, from a sequence (u^k, σ^k) obtained by the algorithm used in [Berga 2012], we can extract a subsequence such that u^k weakly converges in H^1 to u and σ^k weakly converges in L^2 to σ, then (u, σ) is a solution of the problem.

We expect to extend these former results to the non-incremental formulation of the SBEN principle in three directions:

(i) Using the particular expression of the bipotential (for instance for the Armstrong-Frederick model in cyclic plasticity of metals), we expect to prove by suitable arguments of majoration that the sequence (u^k, σ^k) has convergent subsequences.

(ii) Next the objective is, in the same spirit, to study the variational formulations including the unilateral contact with Coulomb's friction.

(iii) Finally, we study the existence properties of the solutions and of convergence of the numerical algorithms for the evolutions problems of the damage and crack propagation by the principle of SBEN [de Saxce 2021]. For the regularized formulations based on the gradient of the damage variable, the Γ-convergence properties of the functional will be used.

References

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[Buliga 2012] Buliga, M, de Saxcé G, Vallée C. A variational formulation for constitutive laws described by bipotentials. *Mathematics and Mechanics of Solids*. 2012 ; 18(1) :78-90.

[Buliga 2016] Buliga, M, de Saxcé G. A symplectic Brezis-Ekeland-Nayroles principle. *Mathematics and Mechanics of Solids*. 2017 ; 22(6).

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[Laborde 2008] Laborde P, Renard, Y. Fixed points strategies for elastostatic frictional contact problems. *Math Meth Appl Sci* 2008 ;31 :415-441.

[Matei 2011] Matei A, Niculescu C. Weak solutions via bipotentials in mechanics of deformable solids.

Candidate:

Highly motivated by applied mathematics to the mechanics, he (she) has a strong background and scientific experience in functional and convex analysis. Skills in nonlinear continuum mechanics are welcome. He (she) appreciates working in a team and is able to interact with other members of the project involved in numerical aspects. He (she) is autonomous and has initiative. He (she) writes and speaks English fluently.